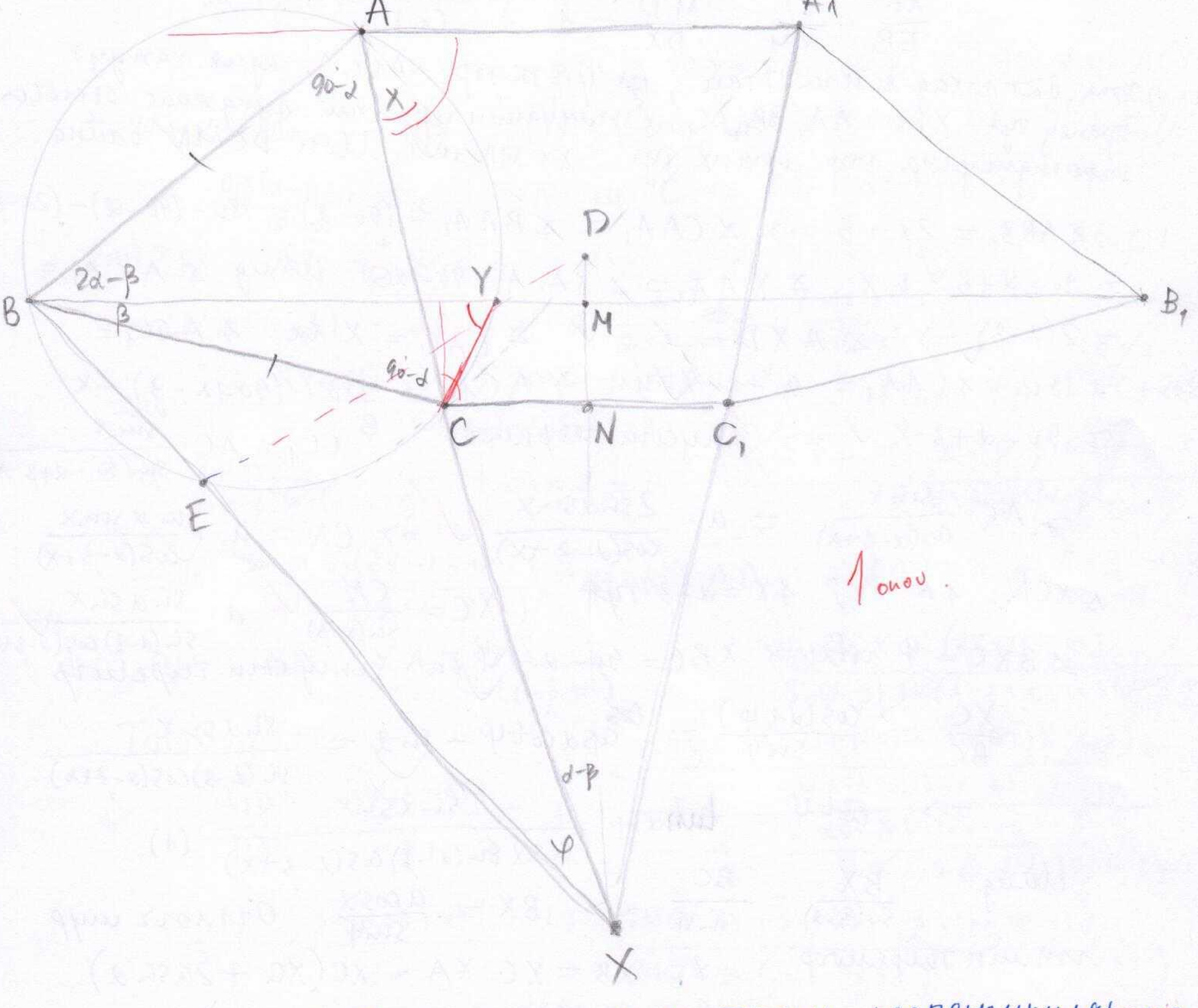
I сорил.

1р сургууль М. Өнөболд

А3.

Нүүр 1



1 оноо.

AA_1, BB_1 ба CC_1 -н дундажид татсан перпендикулярууд давхцах тул $AA_1 \| BB_1 \| CC_1$ ба эдгээр хэрчмүүд хос хосоороо адил хажуут трапец үүсгэнэ (учир нь, BB_1 ба CC_1 -н дундажуудыг M ба N гэвэл $MN \perp BB_1$ ба $BM = B_1M$, $CN = C_1N$ => ~~BE~~ Пифагорын теоремоор $BC^2 = MN^2 + (BM - CN)^2 = MN^2 + (B_1M - C_1N)^2 = B_1C_1^2$ => $BC = B_1C_1$ буюу BCC_1B_1 адил хажуут трапец болно). Иймд ACC_1A_1 тойрог багтана. $P_1 \dots P_n$ тойрог багтах олон өнцөгтийн багтаасан тойргийг $(P_1 \dots P_n)$ гэж тэмдэглэе.

$$\left.\begin{array}{l}(ABC),\ (A_1BC_1) \text{ -н радикал тэнхлэг } BE \\ (ABC),\ (ACC_1A_1) \text{ -н радикал тэнхлэг } AC \\ (A_1BC_1),\ (ACC_1A_1) \text{ -н радикал тэнхлэг } A_1C_1\end{array}\right\} \Rightarrow$$

BE ба AC, A_1C_1 нэг цэгт огтлолцоно. Уг цэгийг X гэе.

Нөхөр 2

$a = AB = BC = A_1B_1 = B_1C_1$ гэе. $\measuredangle BAC = \measuredangle BCA = 90 - \alpha$ гэе, тэ $\measuredangle ABC = 2\alpha$ ба $AC = 2a\sin\alpha$. $\measuredangle CBB_1 = \beta$ гэе, тэгвэл $\measuredangle ABB_1 = 2\alpha - \beta$. $BB_1 \cap (ABC) = \{B, X\}$ гэе. E, X, D нэг шулуун дээр оршино гэж батлъя, Менелайн теоремоор

$$\frac{XE}{EB} \cdot \frac{BX}{XM} \cdot \frac{MD}{DX} = 1 \quad (*)$$

гэж батлахад хангалттай, $\cancel{Д}$ учир нь ACC_1A_1 адил хажуут трапец тул X нь AA_1, BB_1, CC_1 хэрчмүүдийн ерөнхий дундаж(аас) босгосон перпендикуляр дээр орших тул $X \in MN$ юм. Мөн $D \in MN$ байна.

$\measuredangle ABB_1 = 2\alpha - \beta \Rightarrow \measuredangle CAA_1 = \measuredangle BAA_1 - (90 - \alpha) = 180 - (90 - \alpha) - (2\alpha - \beta)$ $= 90 - \alpha + \beta$, ө.х $\measuredangle XAA_1 = \measuredangle XA_1A = 90 - \alpha \overset{+}{-} \beta$. Иймд $\measuredangle AXA_1 =$ $= 2(\alpha - \beta) \Rightarrow \measuredangle AXD = \alpha - \beta$. $\measuredangle CAC_1 = x$ гэе. $\measuredangle ACC_1 =$ $= 180 - \measuredangle CAA_1 = 90 + \alpha - \beta$ тул $\measuredangle AC_1C = 180 - (90 + \alpha - \beta) - x$ $= 90 - \alpha + \beta - x \Rightarrow$ Синусын теоремоор $CC_1 = AC \cdot \dfrac{\sin x}{\sin(90 - \alpha + \beta - x)}$

$$= AC \cdot \frac{\sin x}{\cos(\alpha - \beta + x)} = a \cdot \frac{2\sin\alpha\sin x}{\cos(\alpha - \beta + x)} \Rightarrow CN = a \cdot \frac{\sin\alpha\sin x}{\cos(\alpha - \beta + x)}.$$

$\triangle XCN$: $\measuredangle N = 90$, $\measuredangle X = \alpha - \beta$ тул $XC = \dfrac{CN}{\sin(\alpha - \beta)} = a \cdot \dfrac{\sin\alpha\sin x}{\sin(\alpha - \beta)\cos(\alpha - \beta + x)}$

$\measuredangle BXC = \varphi$ гэвэл $\measuredangle XBC = 90 - \alpha - \varphi$ ба синусын теоремоор

$$\frac{XC}{BC} = \frac{\cos(\alpha + \varphi)}{\sin\varphi} = \cos\alpha\cot\varphi - \sin\alpha = \frac{\sin\alpha\sin x}{\sin(\alpha - \beta)\cos(\alpha - \beta + x)}$$

$$\Rightarrow \cot\varphi = \tan\alpha + \frac{\sin\alpha\sin x}{\cos\alpha\sin(\alpha - \beta)\cos(\alpha - \beta + x)} \quad (1)$$

Иймд $\dfrac{BX}{\sin(90 - \alpha)} = \dfrac{BC}{\sin\varphi} \Rightarrow BX = \dfrac{a\cos\alpha}{\sin\varphi}$. Огтлогч шүр-гэгчийн теоремоор $XE \cdot XB = XC \cdot XA = XC(XC + 2a\sin\alpha)$

$$= a \cdot \frac{\sin\alpha\sin x}{\sin(\alpha - \beta)\cos(\alpha - \beta + x)}\left(a \cdot \frac{\sin\alpha\sin x}{\sin(\alpha - \beta)\cos(\alpha - \beta + x)} + 2a\sin\alpha\right) =$$

$$= a^2 \cdot \frac{\sin\alpha\sin x}{\sin(\alpha - \beta)\cos(\alpha - \beta + x)}\left(\frac{\sin\alpha\sin x}{\sin(\alpha - \beta)\cos(\alpha - \beta + x)} + 2\sin\alpha\right)$$

$$= a^2 \cdot \frac{\sin^2\alpha\sin x}{\sin(\alpha - \beta)\cos(\alpha - \beta + x)} \cdot \frac{\sin x + 2\sin(\alpha - \beta)\cos(\alpha - \beta + x)}{\sin(\alpha - \beta)\cos(\alpha - \beta + x)}$$

$$= a^2 \cdot \frac{\sin^2\alpha\sin x\sin(2\alpha - 2\beta + x)}{\sin^2(\alpha - \beta)\cos^2(\alpha - \beta + x)} = \frac{a\cos\alpha}{\sin\varphi} \cdot XE$$

$$\Rightarrow XE = a \cdot \frac{\sin\varphi\sin^2\alpha\sin x\sin(2\alpha - 2\beta + x)}{\cos\alpha\sin^2(\alpha - \beta)\cos^2(\alpha - \beta + x)}.$$

Иймд $\frac{XE}{EB} = \frac{XE}{XB-XE} = \frac{1}{\frac{XB}{XE}-1} =$ Нүүр 3

$$= \frac{1}{\frac{\cos^2\alpha \sin^2(\alpha-\beta)\cos^2(\alpha-\beta+x)}{\sin^2\varphi \sin^2\alpha \sin x \sin(2\alpha-2\beta+x)} - 1} \quad (2)$$ болно.

$\triangle BYC$: $\angle BYC = \angle BAC = 90° - \alpha$ ба $\angle BCY = 90° - \alpha + \angle ACY$ $= 90° - \alpha + 2\alpha - \beta = 90° + \alpha - \beta$ тул синусын теоремоор $BY = BC \cdot \frac{\sin(90°+\alpha-\beta)}{\sin(90°-\alpha)}$ $= a \cdot \frac{\cos(\alpha-\beta)}{\cos\alpha}$ ба MN нь C-с BB_1 шулуун хүртэлх зай тул $MN = BC\sin\beta = a\sin\beta \Rightarrow XM = MN + XN = a\sin\beta +$ $+ CN\cot(\alpha-\beta) = a\sin\beta + a\frac{\sin\alpha\sin x}{\cos(\alpha-\beta+x)} \cdot \frac{\cos(\alpha-\beta)}{\sin(\alpha-\beta)} =$

$$= a \cdot \frac{\sin\alpha\sin x\cos(\alpha-\beta) + \sin\beta\cos(\alpha-\beta+x)\sin(\alpha-\beta)}{\cos(\alpha-\beta+x)\sin(\alpha-\beta)}$$

Мөн $XA = XC + 2a\sin\alpha$

$$= a \cdot \frac{\sin\alpha\sin x}{\sin(\alpha-\beta)\cos(\alpha-\beta+x)} + 2a\sin\alpha = a \cdot \frac{\sin\alpha\sin x + 2\sin\alpha\sin(\alpha-\beta)\cos(\alpha-\beta+x)}{\sin(\alpha-\beta)\cos(\alpha-\beta+x)}$$

$$= a \cdot \frac{\sin\alpha\sin(2\alpha-2\beta+x)}{\sin(\alpha-\beta)\cos(\alpha-\beta+x)}$$ ба $\triangle XAD$: $\angle XAD = x$, $\angle AXD = \alpha-\beta$

тул $$XD = XA\frac{\sin x}{\sin(\alpha-\beta+x)} = a \cdot \frac{\sin\alpha\sin x\sin(2\alpha-2\beta+x)}{\sin(\alpha-\beta)\sin(\alpha-\beta+x)\cos(\alpha-\beta+x)}$$

Тэгэхээр

$$\frac{MD}{DX} = \frac{XD-XM}{XD} = 1 - \frac{a \cdot \frac{\sin\alpha\sin x\cos(\alpha-\beta) + \sin\beta\cos(\alpha-\beta+x)\sin(\alpha-\beta)}{\cancel{\cos(\alpha-\beta+x)}\cancel{\sin(\alpha-\beta)}}}{a \cdot \frac{\sin\alpha\sin x\sin(2\alpha-2\beta+x)}{\cancel{\sin(\alpha-\beta)}\sin(\alpha-\beta+x)\cancel{\cos(\alpha-\beta+x)}}}$$

$$= 1 - \frac{\sin(\alpha-\beta+x)\left(\sin\alpha\sin x\cos(\alpha-\beta) + \sin\beta\cos(\alpha-\beta+x)\sin(\alpha-\beta)\right)}{\sin\alpha\sin x\sin(2\alpha-2\beta+x)} \quad (3).$$

$\angle BXM = \varphi + \alpha - \beta$ тул $BM = XM \cdot \tan(\varphi+\alpha-\beta) =$

$$= a \cdot \frac{\left(\sin\alpha\sin x\cos(\alpha-\beta) + \sin\beta\cos(\alpha-\beta+x)\sin(\alpha-\beta)\right)\sin(\varphi+\alpha-\beta)}{\cos(\alpha-\beta+x)\sin(\alpha-\beta) \cdot \cos(\varphi+\alpha-\beta)}$$ Тэгэхээр

$$\frac{BY}{YM} = \frac{BY}{BM-BY} = \frac{1}{-1+\frac{BM}{BY}} =$$

$$= \frac{1}{-1 + \frac{\cos\alpha\sin(\varphi+\alpha-\beta)\left(\sin\alpha\sin x\cos(\alpha-\beta) + \sin\beta\cos(\alpha-\beta+x)\sin(\alpha-\beta)\right)}{\cos(\alpha-\beta)\cos(\alpha-\beta+x)\sin(\alpha-\beta)\cos(\varphi+\alpha-\beta)}} \quad (4)$$

Нүүр 4

$$\cancel{\cos(\alpha-\beta)\cos(\alpha-\beta+x)\sin(\alpha-\beta)\cos(\varphi+\alpha-\beta)}$$

$$\cancel{\cos\alpha\sin(\varphi+\alpha-\beta)\ (\sin\alpha\sin x\cos(\alpha-\beta)+\sin\beta\cos(\alpha-\beta+x)\sin(\alpha-\beta))-\cos(\alpha-\beta)\cos(\alpha-\beta+x)\sin(\alpha-\beta)}$$

болно. Одоо, sum-to-product ба product-to-sum ~~томьёо~~ томьёонуудаа

$$\sin\alpha\sin x\cos(\alpha-\beta)+\sin\beta\cos(\alpha-\beta+x)\sin(\alpha-\beta)=$$

$$=\frac{1}{2}\Big(\sin\alpha\big(\sin(\alpha+x-\beta)+\sin(x-\alpha+\beta)\big)+2\sin\beta\cos(\alpha-\beta+x)\sin(\alpha-\beta)\Big)$$

$$=\frac{1}{2}\Big(\sin\alpha\sin(\alpha+x-\beta)+\sin\alpha\sin(x-\alpha+\beta)+2\sin\beta\cos(\alpha-\beta+x)\sin(\alpha-\beta)\Big)$$

$$=\frac{1}{2}\Big(\sin\alpha\sin(\alpha+x-\beta)+\sin\alpha\sin(x-\alpha+\beta)+\sin\beta\sin(2\alpha-2\beta+x)-\sin\beta\sin x\Big)$$

$$=\frac{1}{4}\Big(\cancel{\cos(x-\beta)}-\cos(2\alpha+x-\beta)+\cos(x-2\alpha+\beta)-\cancel{\cos(x+\beta)}+2\sin\beta\sin(2\alpha-2\beta+x)+\cancel{\cos(x+\beta)}-\cancel{\cos(x-\beta)}\Big)$$

$$=\frac{1}{4}\Big(2\sin\beta\sin(2\alpha-2\beta+x)-2\sin x\sin(\beta-2\alpha)\Big)$$

$$=\frac{1}{2}\Big(\sin\beta\sin(2\alpha-2\beta+x)+\sin x\sin(2\alpha-\beta)\Big)$$

$$=\frac{1}{4}\Big(\cos(2\alpha-3\beta+x)-\cos(2\alpha-\beta+x)+\cos(2\alpha-x-\beta)-\cos(2\alpha-\beta+x)\Big)$$

$$=\frac{1}{4}\Big(\cos(2\alpha-3\beta+x)+\cos(2\alpha-x-\beta)-2\cos(2\alpha-\beta+x)\Big)$$

$$=\frac{1}{2}\Big(\cos(2\alpha-2\beta)\cos(x-\beta)-\cos(2\alpha-\beta+x)\Big)\ (☆)$$

гэж задрахыг анхаарвал, мөн sum-to-product ба product-to-sum томьёог

$$\frac{MD}{DX}=1-\frac{\sin(\alpha-\beta+x)\cdot\frac{1}{2}\big(\cos(2\alpha-2\beta)\cos(x-\beta)-\cos(2\alpha-\beta+x)\big)}{\sin\alpha\sin x\sin(2\alpha-2\beta+x)}$$

$$=1-\frac{\sin(\alpha-\beta+x)\big(\cos(2\alpha-2\beta)\cos(x-\beta)-\cos(2\alpha-\beta+x)\big)}{2\sin\alpha\sin x\sin(2\alpha-2\beta+x)}$$

$$=1-\frac{\cos(2\alpha-2\beta)\big(\sin\alpha+\sin(\alpha+2x-2\beta)\big)-2\sin(\alpha-\beta+x)\cos(2\alpha-\beta+x)}{4\sin\alpha\sin x\sin(2\alpha-2\beta+x)}$$

$$=\frac{4\sin\alpha\sin x\sin(2\alpha-2\beta+x)-\cos(2\alpha-2\beta)\sin\alpha-\cos(2\alpha-2\beta)\sin(\alpha+2x-2\beta)+2\sin(\alpha-\beta+x)\cos(2\alpha-\beta+x)}{4\sin\alpha\sin x\sin(2\alpha-2\beta+x)}$$

$$=\frac{2\sin\alpha\big(\cos(2\alpha-2\beta)-\cos(2\alpha-2\beta+2x)\big)-\cos(2\alpha-2\beta)\sin\alpha-\cos(2\alpha-2\beta)\sin(\alpha+2x-2\beta)+2\sin(\alpha-\beta+x)\cos(2\alpha-\beta+x)}{4\sin\alpha\sin x\sin(2\alpha-2\beta+x)}$$

$$= \frac{\cos^2(\alpha-\beta)\sin\alpha\sin(\alpha-\beta+x) - \cos\alpha\sin^2(\alpha-\beta)\cos(\alpha-\beta+x)}{\sin(\alpha-\beta)\cos(\alpha-\beta)\left(\cos\alpha\cos(\alpha-\beta+x)+\sin\alpha\sin(\alpha-\beta+x)\right)} \; ?$$

$$= \frac{\sin\alpha\cos(\alpha-\beta)\left(\sin(2\alpha-2\beta+x)+\sin x\right) - \cos\alpha\sin(\alpha-\beta)\left(\sin(2\alpha-2\beta+x)-\sin x\right)}{2\sin(\alpha-\beta)\cos(\alpha-\beta)\cos(\beta-x)}$$

$$= \frac{\sin x\left(\sin\alpha\cos(\alpha-\beta)+\cos\alpha\sin(\alpha-\beta)\right) + \sin(2\alpha-2\beta+x)\left(\sin\alpha\cos(\alpha-\beta)-\cos\alpha\sin(\alpha-\beta)\right)}{2\sin(\alpha-\beta)\cos(\alpha-\beta)\cos(\beta-x)}$$

$$= \frac{\sin x\sin(2\alpha-\beta)+\sin\beta\sin(2\alpha-2\beta+x)}{2\sin(\alpha-\beta)\cos(\alpha-\beta)\cos(\beta-x)}$$

том гарах тул (4)-2 ? ? ?

$$\frac{BY}{YM} = \frac{1}{-1+\dfrac{\cos\alpha\left(\sin\alpha\sin x\cos(\alpha-\beta)+\sin\beta\cos(\alpha-\beta+x)\sin(\alpha-\beta)\right)}{\cos(\alpha-\beta)\cos(\alpha-\beta+x)\sin(\alpha-\beta)}\cdot\dfrac{1}{\cot(\varphi+\alpha-\beta)}}$$

$$= \frac{1}{-1+\dfrac{\cos\alpha\left(\sin\alpha\sin x\cos(\alpha-\beta)+\sin\beta\cos(\alpha-\beta+x)\sin(\alpha-\beta)\right)}{\cos(\alpha-\beta)\cos(\alpha-\beta+x)\sin(\alpha-\beta)}\cdot\dfrac{2\sin(\alpha-\beta)\cos(\alpha-\beta)\cos(\beta-x)}{\sin x\sin(2\alpha-\beta)+\sin\beta\sin(2\alpha-2\beta+x)}}$$

$$\overset{(*)}{=} \frac{1}{-1+\dfrac{2\cos(\beta-x)\cos\alpha\cdot\frac{1}{2}\left(\cos(2\alpha-2\beta)\cos(x-\beta)-\cos(2\alpha-\beta+x)\right)}{\cos(\alpha-\beta+x)\left(\sin x\sin(2\alpha-\beta)+\sin\beta\sin(2\alpha-2\beta+x)\right)}}$$

~~sum-to-product~~
product-to-sum

$$\frac{1}{-1+\dfrac{2\cos(\beta-x)\cos\alpha\left(\cos(2\alpha-2\beta)\cos(x-\beta)-\cos(2\alpha-\beta+x)\right)}{\cos(\alpha-\beta)\left(\cos(2\alpha-\beta-x)-\cos(2\alpha-\beta+x)+\cos(2\alpha-3\beta+x)-\cos(2\alpha-\beta+x)\right)}}$$

$$= \frac{1}{-1+\dfrac{2\cos(\beta-x)\cos\alpha\left(\cos(2\alpha-2\beta)\cos(x-\beta)-\cos(2\alpha-\beta+x)\right)}{\cos(\alpha-\beta)\left(\cos(2\alpha-\beta-x)+\cos(2\alpha-3\beta+x)-2\cos(2\alpha-\beta+x)\right)}}$$

sum-to-product

$$\frac{1}{-1+\dfrac{2\cos(\beta-x)\cos\alpha}{\cos(\alpha-\beta)}\dfrac{\left(\cos(2\alpha-2\beta)\cos(x-\beta)-\cos(2\alpha-\beta+x)\right)}{\left(2\cos(2\alpha-2\beta)\cos(x-\beta)-2\cos(2\alpha-\beta+x)\right)}}$$

Нурр 5

$$= \frac{2\sin\alpha\cos(2\alpha-2\beta) - 2\sin\alpha\cos(2\alpha-2\beta+2x) - \cancel{\cos(2\alpha-2\beta)\sin x} + 2\sin(\alpha-\beta+x) - \cos(2\alpha-2\beta)\sin(\alpha}{4\sin\alpha\sin x\sin(2\alpha-2\beta+x)}$$

$$= \frac{\cos(2\alpha-2\beta)\left(\sin\alpha - \sin(\alpha+2x-2\beta)\right) - 2\sin\alpha\cos(2\alpha+2\beta+2x) + 2\sin(\alpha-\beta+x)\cos(2\alpha-\beta+x)}{4\sin\alpha\sin x\sin(2\alpha-2\beta+x)}$$

$$= \frac{\cos(2\alpha-2\beta)\cdot 2\cos(\alpha+x-\beta)\sin(\beta-x) - 2\sin\alpha\cos(2\alpha+2\beta+2x) + 2\sin(\alpha-\beta+x)\cdot\cos(2\alpha-\beta+x)}{4\sin\alpha\sin x\sin(2\alpha-2\beta+x)}$$

$$= (3') \frac{\cos(2\alpha-2\beta)\cos(\alpha+x-\beta)\sin(\beta-x) - \sin\alpha\cos(2\alpha+2\beta+2x) + \sin(\alpha+x-\beta)\cos(2\alpha+x-\beta)}{2\sin\alpha\sin x\sin(2\alpha+x-2\beta)}$$

Бално.

$$\frac{BY}{YM} = \frac{\cos(\alpha-\beta)\cos(\alpha-\beta+x)\sin(\alpha-\beta)\cos(\varphi+\alpha-\beta)}{\cos\alpha\sin(\varphi+\alpha-\beta)\cdot\frac{1}{2}\left(\cos(2\alpha-2\beta)\cos(x-\beta) - \cos(2\alpha-\beta+x)\right) - \cos(\alpha-\beta)\cos(\alpha-\beta+x)\sin(\alpha-\beta)\cos(\varphi+\alpha-\beta)}$$

$$= \frac{2\cos(\alpha-\beta)\cos(\alpha-\beta+x)\sin(\alpha-\beta)\cos(\varphi+\alpha-\beta)}{\cos\alpha\sin(\varphi+\alpha-\beta)\cos(2\alpha-2\beta)\cos(x-\beta) - \cos\alpha\sin(\varphi+\alpha-\beta)\cos(2\alpha-\beta+x) - 2\cos(\alpha-\beta)\cos(\alpha-\beta+x)\sin(\alpha-\beta)\cos(\varphi+\alpha-\beta)}$$

$$= \frac{\sin(2\alpha-2\beta)\cos(\alpha-\beta+x)\cos(\varphi+\alpha-\beta)}{\cos\alpha\sin(\varphi+\alpha-\beta)\cos(2\alpha-2\beta)\cos(x-\beta) - \cos\alpha\sin(\varphi+\alpha-\beta)\cos(2\alpha-\beta+x) - \sin(2\alpha-2\beta)\cos(\alpha-\beta+x)\cos(\varphi+\alpha-\beta)}$$

Болно

Мои $\cot\varphi = \tan\alpha + \dfrac{\sin\alpha\sin x}{\cos\alpha\sin(\alpha-\beta)\cos(\alpha-\beta+x)} =$

$$= \frac{\sin\alpha}{\cos\alpha}\cdot\frac{\sin(\alpha-\beta)\cos(\alpha-\beta+x)+\sin x}{\sin(\alpha-\beta)\cos(\alpha-\beta+x)}$$ изгуза

$$\cot(\varphi+\alpha-\beta) = \frac{\cot\varphi\cot(\alpha-\beta)-1}{\cot\varphi+\cot(\alpha-\beta)} = \frac{\cot\varphi\cos(\alpha-\beta)-\sin(\alpha-\beta)}{\cos(\alpha-\beta)+\cot\varphi\sin(\alpha-\beta)}$$

$$= \frac{\cos(\alpha-\beta)\cdot\dfrac{\sin\alpha\left(\sin(\alpha-\beta)\cos(\alpha-\beta+x)+\sin x\right)}{\cos\alpha\sin(\alpha-\beta)\cos(\alpha-\beta+x)} - \sin(\alpha-\beta)}{\cos(\alpha-\beta) + \sin(\alpha-\beta)\cdot\dfrac{\sin\alpha\left(\sin(\alpha-\beta)\cos(\alpha-\beta+x)+\sin x\right)}{\cos\alpha\sin(\alpha-\beta)\cos(\alpha-\beta+x)}}$$

$$= \frac{\cos(\alpha-\beta)\cdot\dfrac{\sin\alpha\cos(\alpha-\beta)\sin(\alpha-\beta+x)}{\cos\alpha\sin(\alpha-\beta)\cos(\alpha-\beta+x)} - \sin(\alpha-\beta)}{\cos(\alpha-\beta) + \sin(\alpha-\beta)\cdot\dfrac{\sin\alpha\cos(\alpha-\beta)\sin(\alpha-\beta+x)}{\cos\alpha\sin(\alpha-\beta)\cos(\alpha-\beta+x)}}$$

$$= \frac{1}{8}\Big((\cos 2\alpha+1)(1-\cos(2\alpha-2\beta))(\cos(2\alpha-2\beta+2x)+1) + (\cos 2\alpha-1)(\cos(2\alpha-2\beta)+1)$$

$$\cancel{(1-\cos(2\alpha-2\beta+2x))}\,(\cos(2\alpha-2\beta+2x)-1) + 4(\cos 2\alpha-1)\sin x \sin(2\alpha-2\beta+x)\Big)$$

$$= \frac{1}{8}\Big(-\cos 2\alpha\cos(2\alpha-2\beta)\cos(2\alpha-2\beta+2x) - \cos 2\alpha\cos(2\alpha-2\beta) + \cos 2\alpha\cos(2\alpha-2\beta+2x)$$

$$-\cos(2\alpha-2\beta)\cos(2\alpha-2\beta+2x) + \cancel{\cos 2\alpha} - \cancel{\cos(2\alpha-2\beta)} + \cancel{\cos(2\alpha-2\beta+2x)}$$

$$+1 + \cos 2\alpha\cos(2\alpha-2\beta)\cos(2\alpha-2\beta+2x) - \cos 2\alpha\cos(2\alpha-2\beta) + \cos 2\alpha\cos(2\alpha-2\beta+2x)$$

$$-\cos(2\alpha+2\beta+2x)\cos(2\alpha-2\beta) - \cancel{\cos 2\alpha} + \cancel{\cos(2\alpha-2\beta)} - \cancel{\cos(2\alpha-2\beta+2x)}$$

$$+1 + 2(\cos 2\alpha-1)(\cos(2\alpha-2\beta)-\cos(2\alpha+2\beta+2x))\Big)$$

$$= \frac{1}{8}\Big(2 + 2(\cos 2\alpha-1)(\cos(2\alpha-2\beta)-\cos(2\alpha+2\beta+2x)) -$$

$$-2\cos 2\alpha\cos(2\alpha-2\beta) - 2\cos(2\alpha-2\beta)\cos(2\alpha-2\beta+2x) + 2\cos 2\alpha\cos(2\alpha-2\beta+2x)$$

$$= \frac{1}{4}\Big(1 + \cos 2\alpha\cos(2\alpha-2\beta) - \cos 2\alpha\cos(2\alpha+2\beta+2x) - \cos(2\alpha-2\beta) +$$

$$+\cos(2\alpha+2\beta+2x) - \cos 2\alpha\cos(\overset{2\alpha-2\beta}{\cancel{2\beta}}) - \cos(2\alpha-2\beta)\overset{\cos}{}(2\alpha+2\beta+2x) + \cancel{2}\cos 2\alpha\cos(2\alpha-2\beta+2x)$$

$$= \frac{1}{4}\Big(1 + \cos(2\alpha+2\beta+2x) - (\cos 2\alpha + \cos(2\alpha-2\beta))\cos(2\alpha+2\beta+2x) +$$

$$+\cos 2\alpha\cos(2\alpha-2\beta+2x) - \cos(2\alpha-2\beta)\Big).$$

Да $\cos(2\alpha-2\beta)\cos(\alpha+x-\beta)\sin(\beta-x) - \sin\alpha\cos(2\alpha+2\beta+2x) + \sin(\alpha+x-\beta)\cdot$

$\cdot\cos(2\alpha+x-\beta) = \cancel{1+\cos 2\alpha+\cos 2\beta+}\ \big[1+\cos(2\beta+2x+2x) - (\cos 2\alpha+\cos(2\alpha-2\beta))$

$\cos(2\alpha+2\beta+2x) + \cos 2\alpha\cos(2\alpha-2\beta+2x) - \cos(2\alpha-2\beta)\big]\cdot \dfrac{\cos(\beta-x)\cos\alpha - \cos(\alpha-\beta)}{\sin\alpha\cos(\alpha-\beta)}$

[illegible]

Нүүр 7

$$= \frac{1}{-1+\frac{\cos(\beta-x)\cos\alpha}{\cos(\alpha-\beta)}} = \frac{\cos(\alpha-\beta)}{\cos(\beta-x)\cos\alpha-\cos(\alpha-\beta)} \quad (4')$$

гэж бичиж болно. Бас, $\frac{1}{\sin^2\varphi} = 1+\cot^2\varphi = 1+\left(\frac{\sin\alpha(\sin(\alpha-\beta)\cos(\alpha-\beta+x)+\sin x}{\cos\alpha\sin(\alpha-\beta)\cos(\alpha-\beta+x)}\right)$

$$= 1+\left(\frac{\sin\alpha\cos(\alpha-\beta)\sin(\alpha-\beta+x)}{\cos\alpha\sin(\alpha-\beta)\cos(\alpha-\beta+x)}\right)^2 = \frac{\cos^2\alpha\sin^2(\alpha-\beta)\cos^2(\alpha-\beta+x)+\sin^2\alpha\cos^2(\alpha-\beta)\sin^2(\alpha-\beta+x)}{\cos^2\alpha\sin^2(\alpha-\beta)\cos^2(\alpha-\beta+x)}$$

үүнээс (2)-д

$$\frac{XE}{EB} = \frac{1}{\frac{\cos^2\alpha\sin^2(\alpha-\beta)\cdot\cos^2(\alpha-\beta+x)}{\sin^2\alpha\sin x\sin(2\alpha-2\beta+x)}\cdot\frac{\cos^2\alpha\sin^2(\alpha-\beta)\cos^2(\alpha-\beta+x)+\sin^2\alpha\cos^2(\alpha-\beta)\sin^2(\alpha-\beta+x)}{\cos^2\alpha\sin^2(\alpha-\beta)\cos^2(\alpha-\beta+x)}-1}$$

$$= \frac{1}{\frac{\cos^2\alpha\sin^2(\alpha-\beta)\cos^2(\alpha-\beta+x)+\sin^2\alpha\cos^2(\alpha-\beta)\sin^2(\alpha-\beta+x)}{\sin^2\alpha\sin x\sin(2\alpha-2\beta+x)}-1}$$

$$= \frac{\sin^2\alpha\sin x\sin(2\alpha-2\beta+x)}{\cos^2\alpha\sin^2(\alpha-\beta)\cos^2(\alpha-\beta+x)+\sin^2\alpha\cos^2(\alpha-\beta)\sin^2(\alpha-\beta+x)-\sin^2\alpha\sin x\sin(2\alpha-2\beta+x)}$$

гэж бичиж болно. (2') ба (3')-д орлуулбал

$$\frac{XE}{EB}\cdot\frac{MD}{DX} = \frac{\sin\alpha\left(\cos(2\alpha-2\beta)\cos(\alpha+x-\beta)\sin(\beta-x)-\sin\alpha\cos(2\alpha+2\beta+2x)+\sin(\alpha+x-\beta)\cos(2\alpha+x-\beta)\right)}{2\left(\cos^2\alpha\sin^2(\alpha-\beta)\cos^2(\alpha-\beta+x)+\sin^2\alpha\cos^2(\alpha-\beta)\sin^2(\alpha-\beta+x)-\sin^2\alpha\sin x\sin(2\alpha-2\beta+x)\right)}$$

$$\cancel{= \sin\alpha\left(\cos(2\alpha-2\beta)-\frac{1}{2}(\sin\alpha+\sin(\alpha+2x-2\beta))\right)-\sin\alpha\cos(2\alpha+2\beta+2x)+}$$

болох ба $\cos^2\alpha\sin^2(\alpha-\beta)\cos^2(\alpha-\beta+x)+\sin^2\alpha\cos^2(\alpha-\beta)\sin^2(\alpha-\beta+x)-$

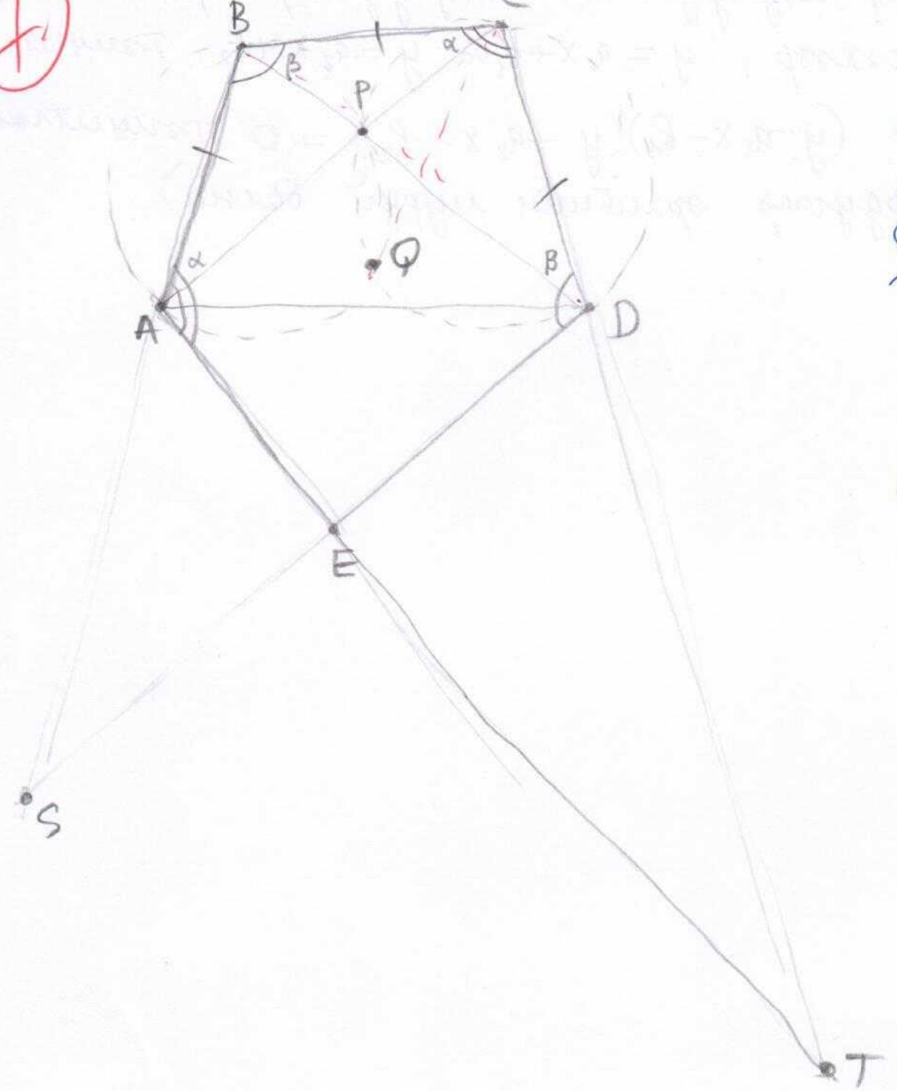
$$-\sin^2\alpha\sin x\sin(2\alpha-2\beta+x) = \frac{\cos2\alpha+1}{2}\cdot\frac{1-\cos(2\alpha-2\beta)}{2}\cdot\frac{\cos(2\alpha-2\beta+2x)+1}{2}$$

$$+\frac{1-\cos2\alpha}{2}\cdot\frac{\cos(2\alpha-2\beta)+1}{2}\cdot\frac{1-\cos(2\alpha-2\beta+2x)}{2}-\frac{1-\cos2\alpha}{2}\sin x\sin(2\alpha-2\beta+x)$$

$$\cancel{=\frac{1}{8}\left((\cos2\alpha+1)(1-\cos(2\alpha-2\beta))(\cos(2\alpha-2\beta+2x)-1)+(1-\cos2\alpha)(\cos\right.}$$

1р сургууль М. Өнөболд В1

7



$AC \cap BD = P$ гэе.

ABP ба DCP-г багтаасан тойргууд хоёр дахь удаагаа Q цэгт огтлолцдог гэе. $\angle ABC = \angle CDE = 180^\circ - 2\alpha$ ба $\angle EAB = \angle BCD = 180^\circ - 2\beta$ гэе. $AB = BC = CD$ учраас $\angle BAC = \angle BCA = \alpha$ ба $\angle CBD = \angle CDB = \beta$ болно. $\angle QAB = \angle QPD = \angle QCD$ ба $\angle QBA = \angle QPA = \angle QDC$ гэдгээс $\triangle QAB \sim \triangle QCD$. Бас $AB = CD$ тул $\triangle QAB = \triangle QCD$ болно, ө.х $QA = QC$ ба $QB = QD$. $AB = BC$ ба $QA = QC$ тул BQ нь AC-н дунджаас босгосон перпендикуляр шулуун болно. Адилхнаар CQ нь BD-н дунджаас босгосон перпендикуляр, иймээс $BQ \perp CA$, $CQ \perp BD$. Тэгэхээр $BP \perp CQ$, $BQ \perp CP \Rightarrow$ P нь $\triangle BQC$-н ортотов, иймд $PQ \perp BC$.

Одоо $E \in PQ$ гэж батлахад хангалттай (эндээс $PE \perp BC$ гэж гарна). $AB \cap DE = S$ ба $AE \cap CD = T$ гэе. $SBCD$ хувьд $\angle SBC = \angle SDC$ ба $\angle CBD = \angle CDB \Rightarrow \angle SBD = \angle SDB$, иймд $SB = SD \Rightarrow$ S нь BD-н дундаж дээр оршино, ө.х $S \in CQ$. Адилхнаар $T \in BQ$, ө.х $Q = BT \cap CS$. Одоо нэг шулуун дээр орших (B, A, S) ба (C, D, T) хос гурвалын хувьд

$$\left.\begin{array}{l} AC \cap BD = P \\ BT \cap CS = Q \\ AT \cap SD = E \end{array}\right\} \Rightarrow$$ Паппын теоремоор P, Q, E нэг шулуун дээр оршино. Бодлого бодогдов (Паппын теорем нь

Паскалийн теоремийн ерөнхий тохиолдлоос гарна, учир нь хавтгай дээрх хоёр шулуун нь хоёрдугаар эрэмбийн муруй үүсгэнэ. Чнээхээр, $y = a_1 x + b_1$ ба $y = a_2 x + b_2$ тэгшит-гэлтэй хоёр шулуун $(y - a_1 x - b_1)(y - a_2 x - b_2) = 0$ тэгшитгэ-ээр үүсгэгдэж хоёрдугаар эрэмбийн муруй болно).

1р сургууль М. Онорболд В2

$p|10$ бол А тоглогч эхний үйлдлээрээ $i=0$ дугаарыг $a_i=a_0=0$ гэж сонгоод, цаашид яаж ч тоглосон гэсэн ялна.

Одоо $(p,10)=1$ тохиолдлыг бодъё. $p\geq 3$ болно, ө.х $p\equiv 1(2)$.

Эхлээд $p\leq 10$ үед хожих стратегитэй болохыг харуулъя. А эхний үйлдлээрээ 0 дугаарыг $a_0=0$ тай хамт сонгоод, цаашид $\{1,2,\dots,p-1\}$ буюу үлдсэн дугааруудын олонлогоо $\{1,2\},\{3,4\},\dots,\{p-2,p-1\}$ гэж хуваая. В хүн i дугаарыг сонговол А хүн i тай хамт нэг олонлогт орох $i\pm 1$ дугаарыг сонгох стратеги баримтална. А хүн j ($j=i-1 \vee i+1$) дугаарыг сонгосон бол, $10^i\cdot a_i+10^j\cdot a_j \vdots p$ байхаар $a_j\in\{0,1,\dots,9\}$ сонгож чадна, учир нь $(10^j,p)=1$ ба a_j-г 10 янзаар сонгож чадах тул, $p\leq 10$ учраас чадна. Иймд, үйлдэл бүрийн дараа $10^{2i-1}\cdot a_{2i-1}+10^{2i}\cdot a_{2i}\vdots p$ байх тул $M\vdots p$.

Одоо $p\geq 11$ тохиолдлыг бодъё. $\mathrm{mod}\,p$-р анхны язгуур g байг. $10\equiv g^k(p)$, $1\leq k\leq p-2$ гэж бичье, учир нь $(10,p)=1$ ба $10\not\equiv 1(p)$ тул ингэж бичиж чадна. Дараах хоёр тохиолдлыг авч үзье.

① $v_2(k)\leq v_2\left(\frac{p-1}{2}\right)$ гэе. $k=2^\alpha\cdot d$, $\alpha\geq 0$ бүхэл ба d сондгой натурал тоо гэж бичье. $p-1\vdots 2^{\alpha+1}$ болно. Иймээс $(p-1)d\vdots 2k$

Тэгэхээр $10^{\frac{p-1}{2^{\alpha+1}}}\equiv g^{k\cdot\frac{p-1}{2^{\alpha+1}}}\equiv g^{\frac{p-1}{2}\cdot d}\equiv -1\ (p)$ болно, учир нь $g^{\frac{p-1}{2}}\equiv -1\ (p)$ ба d сондгой билээ. Одоо $\{1,2,\dots,p-1\}$ олонлогийг

$\left\{1,\ 1+\frac{p-1}{2^{\alpha+1}},\ 1+2\cdot\frac{p-1}{2^{\alpha+1}},\ \dots,\ 1+(2^{\alpha+1}-1)\cdot\frac{p-1}{2^{\alpha+1}}\right\}$

$\left\{2,\ 2+\frac{p-1}{2^{\alpha+1}},\ 2+2\cdot\frac{p-1}{2^{\alpha+1}},\ \dots,\ 2+(2^{\alpha+1}-1)\cdot\frac{p-1}{2^{\alpha+1}}\right\}$

$\vdots$

$\left\{\frac{p-1}{2^{\alpha+1}},\ \frac{p-1}{2^{\alpha+1}}+\frac{p-1}{2^{\alpha+1}},\ \frac{p-1}{2^{\alpha+1}}+2\cdot\frac{p-1}{2^{\alpha+1}},\ \dots,\ \frac{p-1}{2^{\alpha+1}}+(2^{\alpha+1}-1)\cdot\frac{p-1}{2^{\alpha+1}}\right\}$

гэж хуваая. Олонлог бүр $2^{\alpha+1}\vdots 2$ ш элементтэй. Олонлог бүрийг "хосуудад", хос бүрийн зөрүү $\frac{p-1}{2^{\alpha+1}}$ байхаар хувааж болно. А тоглогч эхлээд $i=0$ дугаарыг $a_i=0$ тоотой хамт сонгоно. Цаашид, В тоглогч ямар нэг i дугаар сонгосон бол А тоглогч i-тай хамт хос үүсгэх нөгөө дугаарыг сонгох стратеги барилтална. Тэгээд, А тоглогч j-г сонгосон бол $a_j=a_i$ гэж сонгоё. $j=i\pm\frac{p-1}{2^{a+1}}$ тул $10^i\cdot a_i+10^j\cdot a_j=a_i(10^i+10^j)$ $=a_i\,10^{\min\{i,j\}}(10^{\frac{p-1}{2^{a+1}}}+1)\vdots p$ болно. Иймд, дээрх стратегиар, $\{i,j\}$ нь хос үүсгэдэг бол $10^i\cdot a_i+10^j a_j\vdots p$ ба иймд $M\vdots p$ болж байна.

② $v_2(k)>v_2(\frac{p-1}{2})$ гэе. Эндээс $v_2(k)\geq v_2(\frac{p-1}{2})+1=$ $=v_2(p-1)$ гэж гарна. $p-1=2^a\cdot d$, d сондгой гэж бичье. $k\vdots 2^a$ ба $a\in\mathbb{N}$. $\{1,2,\dots,p-1\}$ олонлогийг

$\{1,1+d,1+2d,\dots,1+(2^a-1)d\}$

$\{2,2+d,2+2d,\dots,2+(2^a-1)d\}$

$\vdots$

$\{d,d+d,d+2d,\dots,d+(2^a-1)d\}$

7 оноо

гэж олонлогуудад хуваах болно. Олонлог бүр 2^a буюу тэгш элементтэй тул олонлог бүрийг ялгавар нь d байх "хос"-уудад хуваах болно. ~~А тоглогч~~ Мөн $10^d\equiv g^{kd}=g^{(p-1)\cdot\frac{k}{2^a}}\equiv 1\ (p)$ болохыг тэмдэглэе. А тоглогч эхлээд $i=0$ дугаарыг $a_i=0$-тай хамт сонгоё. Цаашид В тоглогч ямар нэг i дугаар сонгосон бол А тоглогч i-тай хамт хос үүсгэх нөгөө дугаар (j болгоё)-ийг сонгох стратеги барилтална. $j=i\pm d$. Бас $a_j=9-a_i$ гэж сонгоё. Тэгвэл $10^i a_i+10^j a_j=10^i a_i+10^j(9-a_i)$ $=9\cdot10^j+a_i(10^i-10^j)$ ба 10^i-10^j нь $j=i-d$ бол $10^{i-d}(10^d-1)\vdots p$, $j=i+d$ бол $10^i(1-10^d)\vdots p$ тул $10^i a_i+10^j a_j\equiv 9\cdot10^j\equiv 9\cdot10^i\ (p)$ болно. Иймд $M\equiv 9\cdot\sum_{\substack{i\text{ нь}\\ \text{хосын ХБ}\\ \text{тоо}}}10^i\ (p)$ гэж бичиж чадна, учир нь $10^0\cdot a_0\vdots p$ ба $\{i,j\}$ хос бол $10^i a_i+10^j a_j\equiv 9\cdot10^i\equiv 9\cdot10^j$ (mod p). Тэгэхээр $M\equiv 9\cdot(10^1+\dots+10^d+10^{1+2d}+\dots+10^{d+2d}+\dots+10^{1+(2^a-1)d}+\dots+10^{d+(2^a-1)d})$ (mod p) ба $10^k+\dots+10^{k+d-1}=10^k\cdot\frac{10^d-1}{9}\vdots p$ тул $M\vdots p$. Бодлого бодогдов.

1р аргүүль М. Онобалд В3.

$T = S$ бол, $f(S) = T$ тодорхойлолтоос $f(S) = S \Rightarrow f(T) = T$ болж батлагдана.

Одоо $T \subset S$ гэе, о.х $S \setminus T$ олонлог хоосон биш. T хоосон биш байх нь ойлгомжтой. $f(T) = T$ гэж батлахын тулд, $f: T \to T$ -г биектив гэж батлахад хангалттай.

Лемма. $\forall a \notin T$ ба $\forall b \in S$: $f(f(f(a))) \neq f(f(b))$ (энд $b \neq f(a)$).

Баталгаа. $g: S \to S$ нь

$$g(x) = \begin{cases} f(x), & x \neq a \\ b, & x = a \end{cases}$$

гэж тодорхойлогдсон гэе. Өгсөн нөхцлөөр $\exists x \in S$: $f(g(f(x))) \neq g(f(g(x)))$. Энд $f(g(x)) \neq a$ (учир нь $a \notin T$) ба $f(x) \neq a$ тул $g(f(g(x))) = f(f(g(x)))$ ба $g(f(x)) = f(f(x))$ болохыг санавал $f(f(f(x))) \neq f(f(g(x)))$. Эндээс $x = a$, учир нь $x \neq a$ бол $f(x) = g(x)$. Тэгэхээр $f(f(f(a))) \neq f(f(g(a))) = f(f(b))$ болж лемма батлагдав.